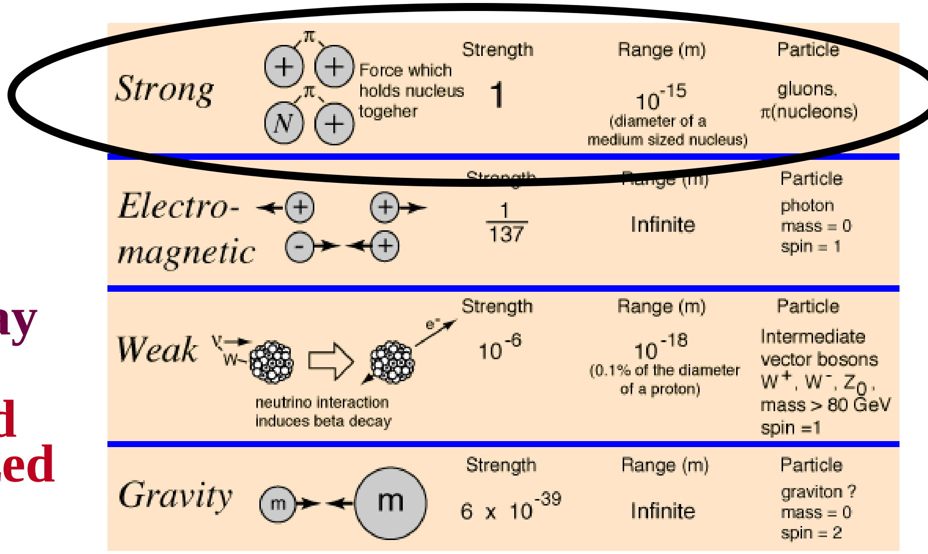
RDCH 702: Lecture 5 Nuclear Force, Structure and Models

- Readings:
  - Nuclear and Radiochemistry: Chapter 10 (Nuclear Models)
  - Modern Nuclear Chemistry: Chapter 5 (Nuclear Forces) and Chapter 6 (Nuclear Structure)
- Characterization of strong force
- Charge Independence
  - Introduce isospin
- Nuclear Potentials
- Simple Shell Model (Focus of lecture)

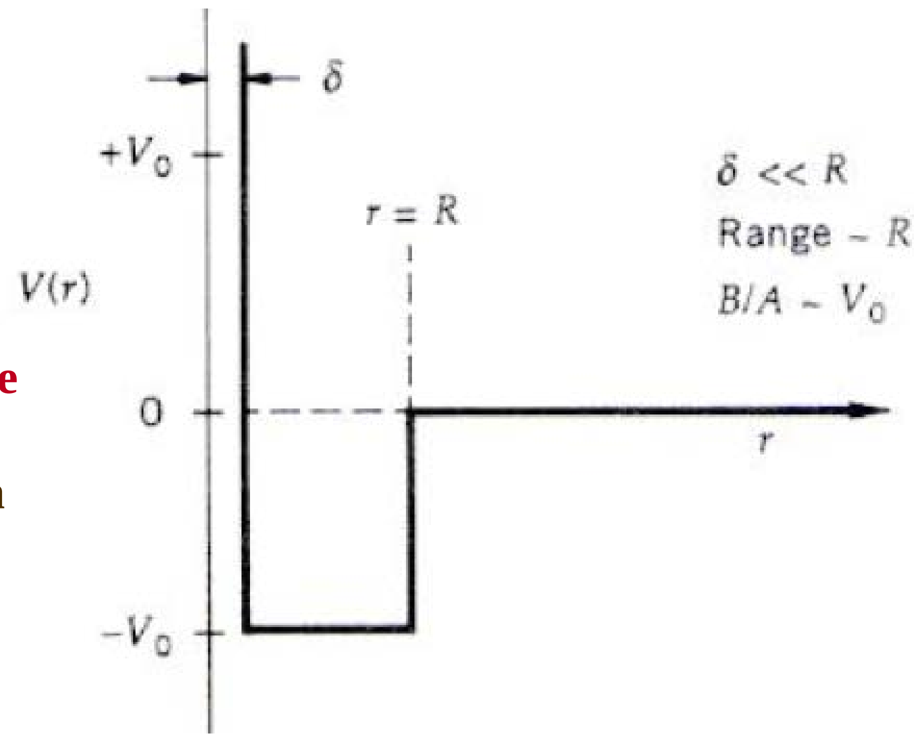


Nuclear Force

- For structure, reactions and decay of nuclei
  - electromagnetic, strong and weak interactions are utilized
- Fundamental forces exhibit exchange character
  - operate through virtual exchange of particles that act as force carriers

Strong Force

- Nuclear force has short range
  - Range of a nucleon
- Nuclear force is strongly attractive and forms a dense nucleus
- Nuclear force has a repulsive core
  - Below a distance (0.5 fm) nuclear force becomes repulsive
- Force between two nucleons has two components
  - spherically symmetric central force
  - asymmetric tensor force
    - → Spin dependent force between nucleons
- Consider ^{2}H
  - Proton and neutron
    - → Parallel spin ^{3}S
      - * Can be in excited state, ^{3}D
      - * Antiparellel is unbound ^{1}S



Charge Independent Force

- Strong force not effected by charge
  - np, nn, pp interactions the same
    - → Electromagnetic force for charge
- Strong force examined by
  - Nucleon-nucleon scattering
  - Mirror nuclei
    - → Isobars with number of p in one nuclei equals number of n in other
    - → Similar energy for net nuclear binding energy
      - * Normalize influence of Coulomb Energy
- Shows proton and neutron two states of same particle

A	Nucleus	Total Binding Energy (MeV)	Coulomb Energy (MeV)	Net nuclear binding energy (MeV)
3	^{3}H	-8.486	0	-8.486
	^{3}He	-7.723	0.829	-8.552
13	^{13}C	-97.10	7.631	-104.734
	^{13}N	-94.10	10.683	-104.770
23	^{23}Na	-186.54	23.13	-209.67
	^{23}Ne	-181.67	27.75	-209.42
41	^{41}Ca	-350.53	65.91	-416.44
	^{41}Sc	-343.79	72.84	-416.63

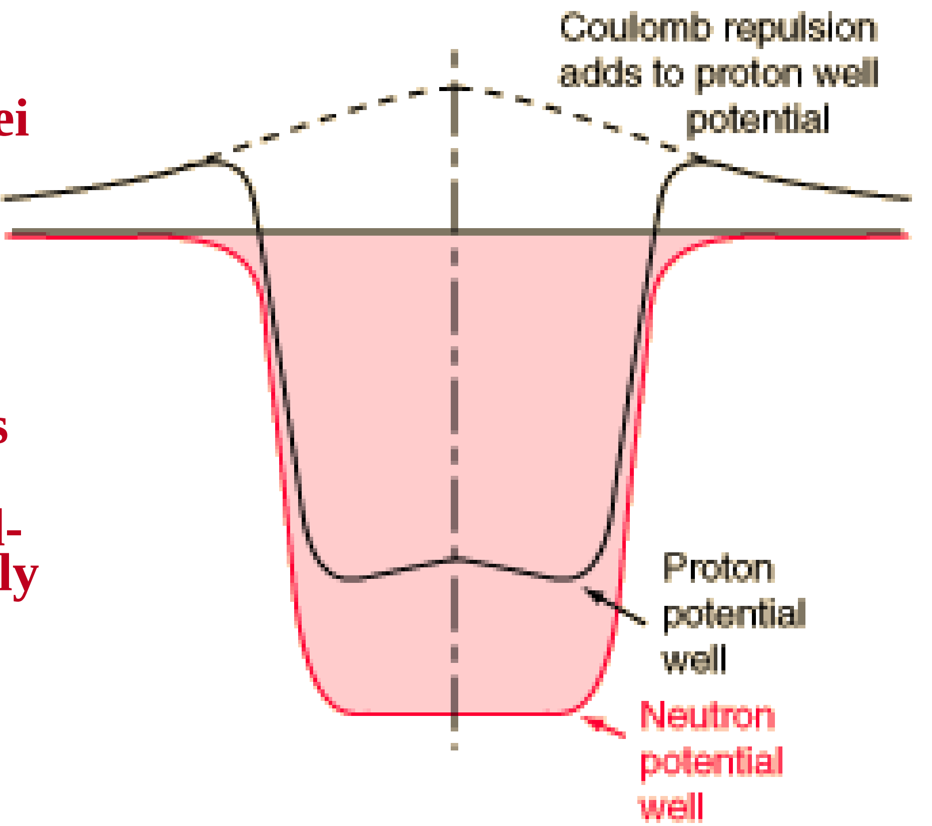
- Isospin is conserved in processes involving the strong interaction
- Isospin forms basis for selection rules for nuclear reactions and nuclear decay processes
- Property of nucleon
  - Analogy to angular momentum
  - T=1/2 for a nucleon
    - → +1/2 for proton, -1/2 for neutron

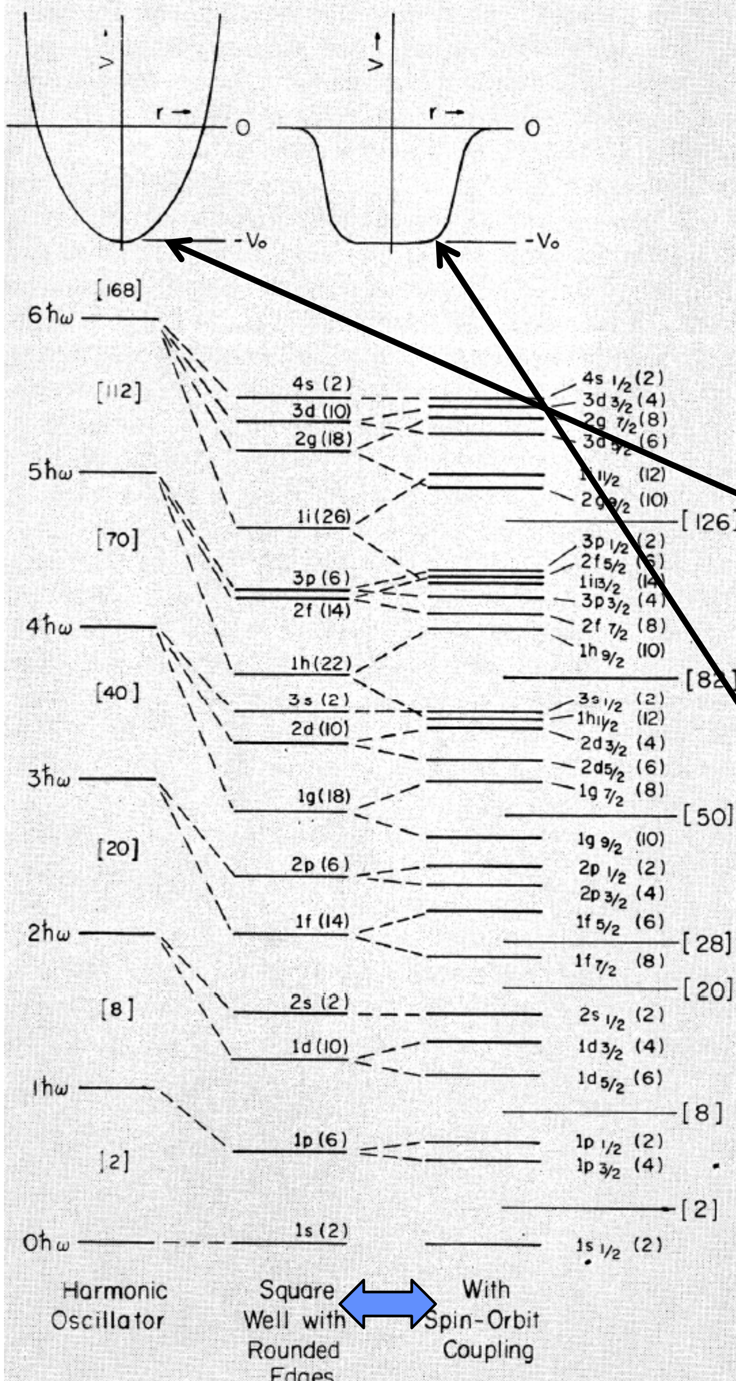
Nuclear Potential Characteristics

- Particles in a potential well
  - Nuclear forces describe potential
  - Small well
  - Well stabilizes nucleons
    - → Free neutrons decay
      - * Neutrons can be stable in nuclear well
    - → Mixture of nucleons stable
      - * 2 protons (^{2}He) unstable
      - * 2 neutrons unstable
    - → A=3
      - * Mixture of n and p stable
        - ➢ 3 protons unstable
- Nuclear force acts between nucleons in uniform way
  - Protons have additional Columbic repulsion that destabilize proton-rich nuclei
    - → Very neutron-rich nuclei are also unstable
  - Light, symmetric nuclei (Z=N) are favored
  - Nuclear force depends on the spin alignment of nucleons
- Potential energy of two nucleons shows similarity to chemical bond potential-energy function

Shell Model

- Interactions among nucleons in nucleus replaced by potential-energy well within which each particle moves freely
- Properties determined by shape of potential energy well
- Experimental Evidence to support model
  - ground-state spin of 0 for all nuclei with even neutron and proton number
  - Magic number for nuclei
  - Systematics of ground-state spins for odd-mass-number nuclei
  - Dependence of magnetic moments of nuclei upon their spins
  - Properties of ground states of odd-mass-number nuclei approximately from odd, unpaired nucleon
    - →All other nucleons provide potential-energy field
    - →determines single-particle quantum states for unpair nucleon
- Stability of nuclei based on number of neutrons and protons



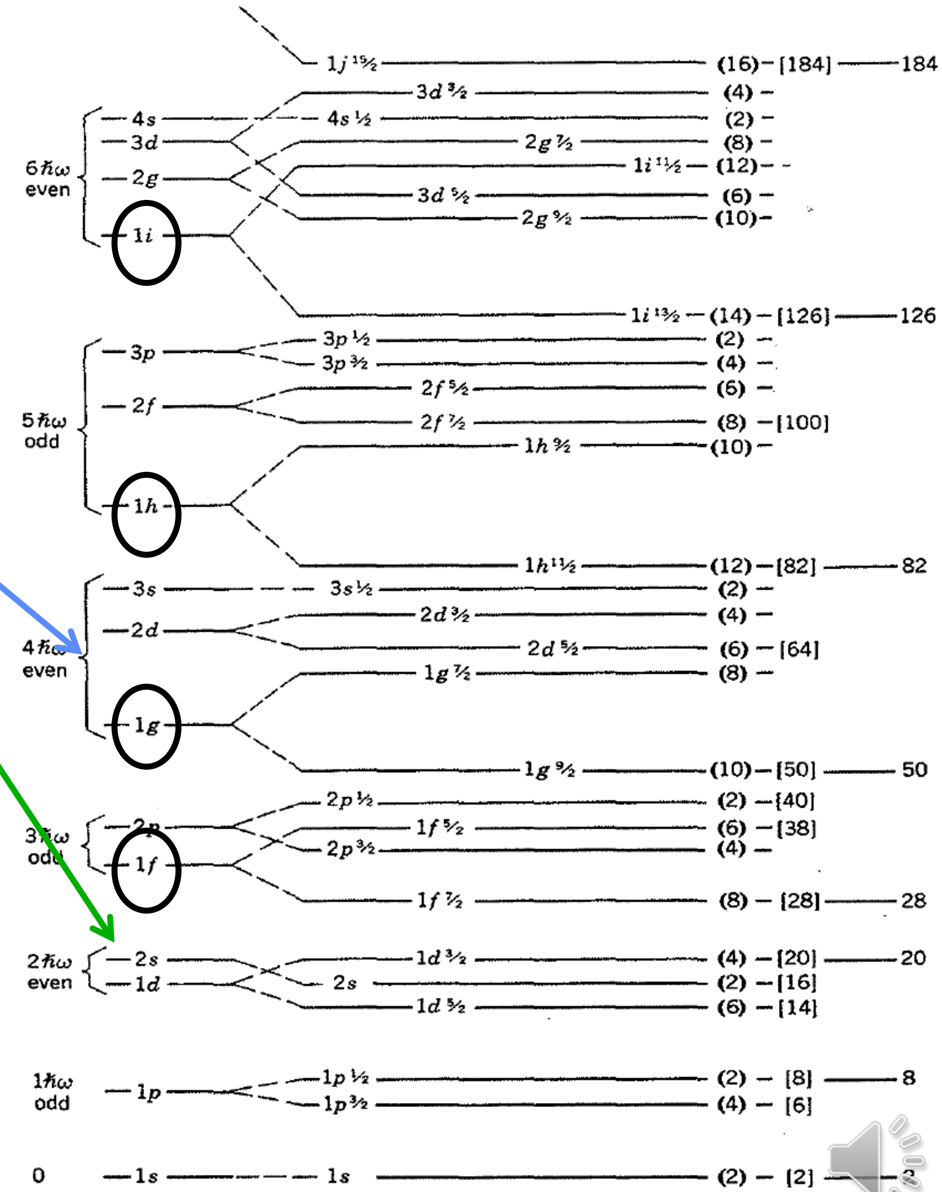


Shell Model

- Model nucleus as a spherical rigid container
  - ▪ square-well potential
    - → potential energy assumed to be zero when particle is inside walls
    - → Particle does not escape
- Harmonic oscillator potential
  - ▪ parabolic shape
  - ▪ steep sides that continue upwards
    - → useful only for the low-lying energy levels
    - → equally spaced energy levels
      - * Potential does not “saturate”
      - * not suitable for large nuclei
- Change from harmonic oscillator to square well lowers potential energy near edge of nucleus
  - ▪ Enhances stability of states near edge of nucleus
  - ▪ States with largest angular momentum most stabilized

Shell Model

- Shell filling
  - States defined by n and l
    - → 1s, 1p, 1d, …
      - * Compare with electrons
  - States with same 2n+l degenerate with same parity (compose level)
    - → 2s = 2*2+0=4
    - → 1d = 2*1+2 =4
    - → 1g=2*1+4=6
    - → 2d=2*2+2=6
    - → 3s=2*3+0=6
- Spin-Orbit Interaction
  - Addition of spin orbit term causes energy level separation according to total angular momentum (j=ℓ+s)
    - → For p, l=1
      - * s=±1/2
      - * j= 1+1/2=3/2 and 1-1/2=1/2
      - * split into fourfold degenerate $1p_{3/2}$ and twofold degenerate $1p_{1/2}$ states
    - → For g, l=4, j=7/2 and 9/2
  - states with parallel coupling and larger total angular momentum values are favored
  - closed shells 28, 50, 82, and 126
    - → splitting of the 1f, 1g, 1h, and 1i
- Each principal quantum number level is a shell of orbitals
- Energy gap between shell the same

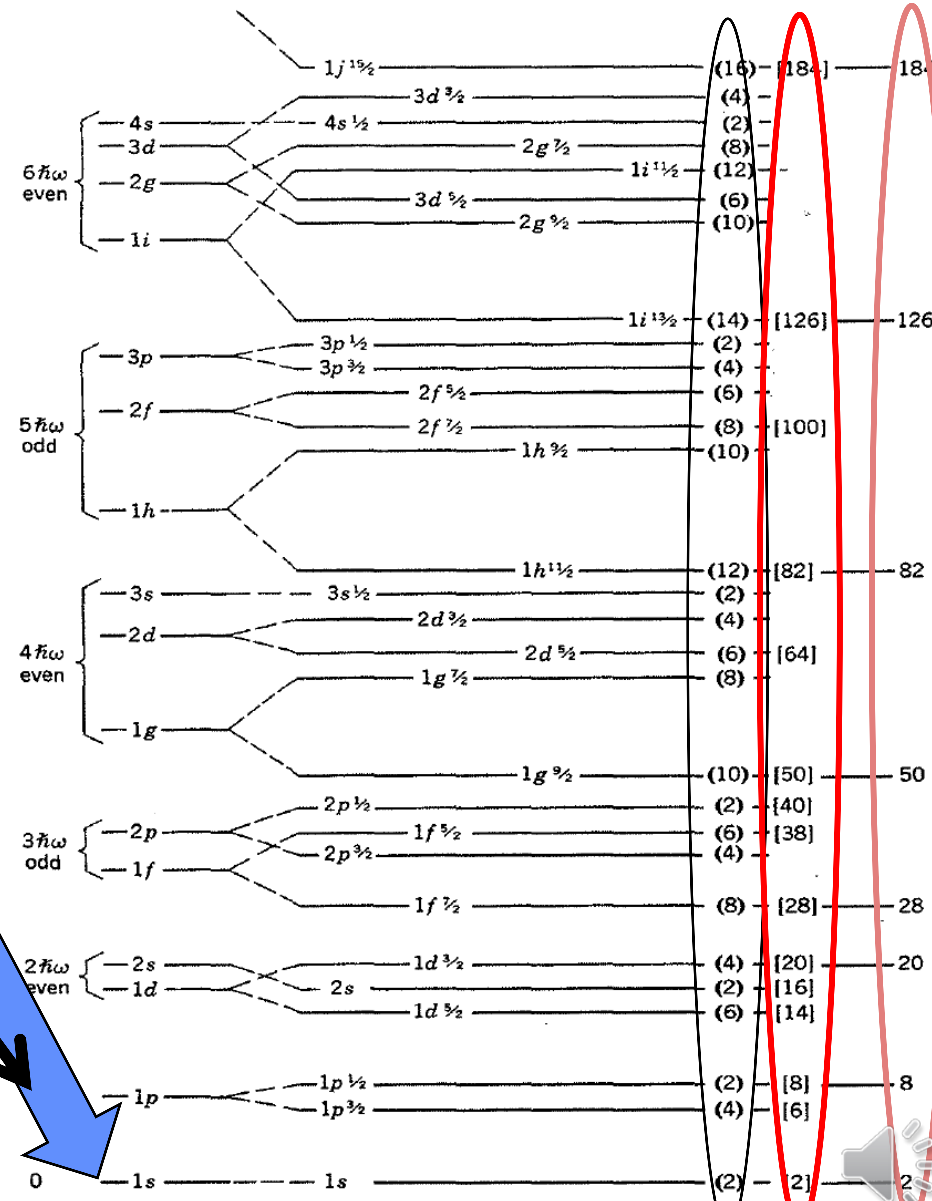


Filling Shells

- **Odd-A Nuclei**
  - **In odd A nucleus of all but one of nucleons considered to have their angular momenta paired off**
    - **→ forming even-even core**
    - **→ single odd nucleon moves essentially independently in this core**
    - **→ net angular momentum of entire nucleus determined by quantum state of single odd nucleon**
      - *** Spin of spin of state, parity based on orbital angular momentum**
        - **Even (s, d, g, i,….)**
        - **Odd (p, f, h,….)**
- **Configuration Interaction**
  - **For nuclides with unpaired nucleons number half way between magic numbers nuclei single-particle model is oversimplification**
    - **→ Contribution from other nucleons in potential well, limitation of model**
- **Odd-Odd Nuclei**
  - **one odd proton and one odd neutron each producing effect on nuclear moments**
  - **No universal rule can be given to predict resultant ground state**
- **Level Order**
  - **applied independently to neutrons and protons**
  - **proton levels increasingly higher than neutron levels as Z increases**
    - **→ Coulomb repulsion effect**
  - **order given within each shell essentially schematic and may not represent exact order of filling**
- **Ground States of Nuclei**
  - **filled shells spherically symmetric and have no spin or orbital angular momentum and no magnetic moment**
  - **ground states of all even-even nuclei have zero spin and even parity**
    - **→ increased binding energy of nucleons**

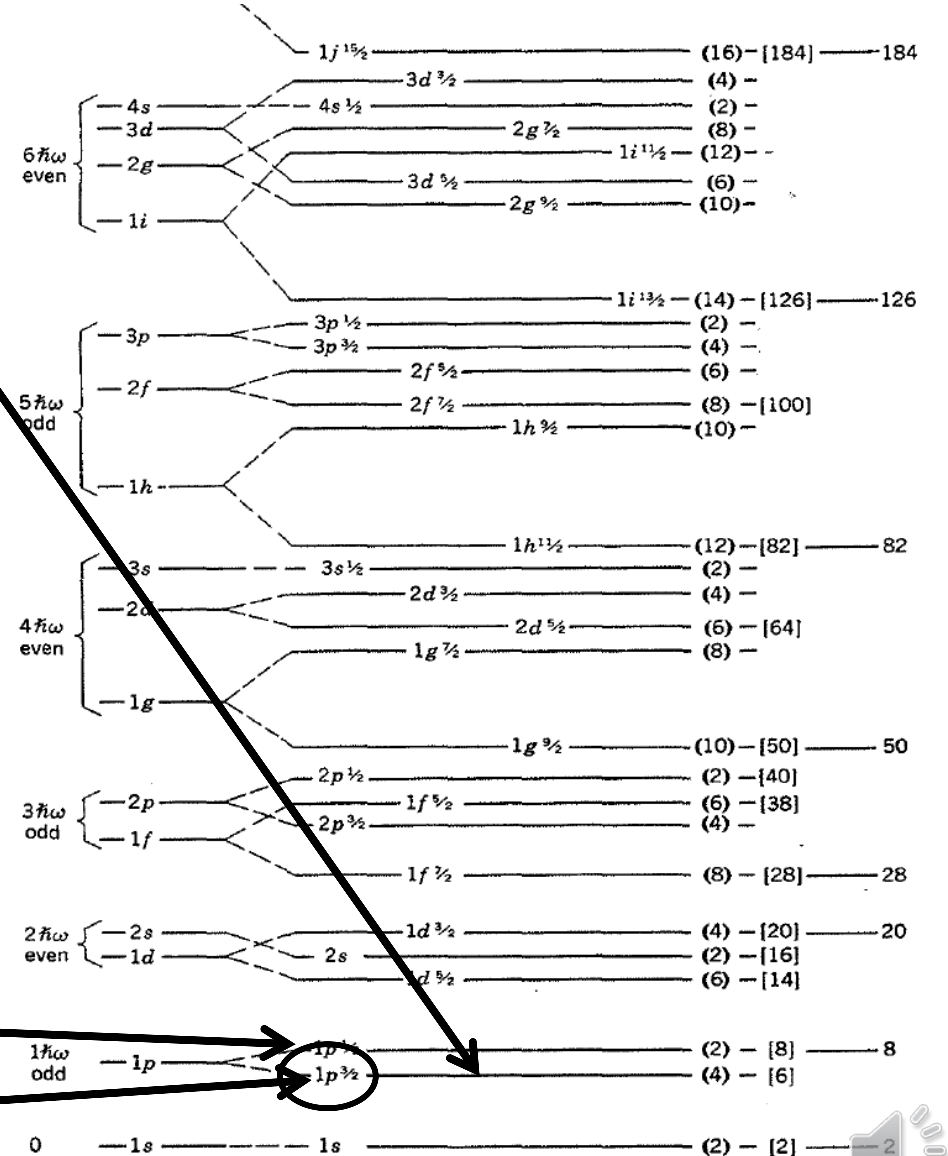
Filling Shells

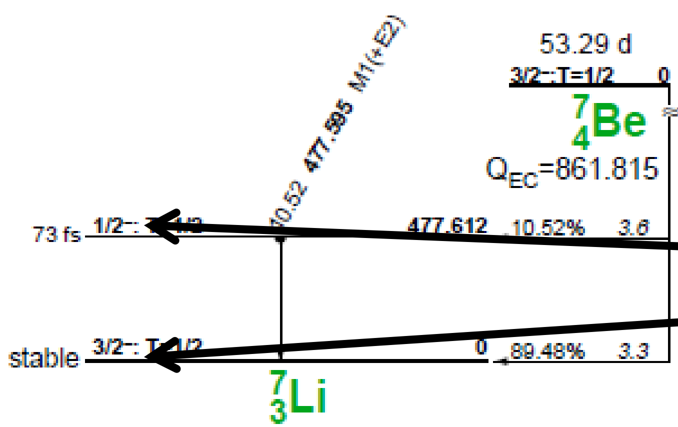
- lowest level is $1s_{1/2}$,
  - s since ℓ=0, ℓ+s=1/2
  - level has on ℓ+1=1 m-value
  - hold only 2 p ons in proton well and two neutrons i utron well
- next levels are $1p_{3/2}$ and 2 pair
  - N=1 ħω
- ^{4}He exact filling of both N= rmonic oscillator shells for neutrons and prot
  - expected to have an e nced stability
- Consider shell filling when N=0 and N=1 ħ ω shells filled
  - eight protons and eight ne ons
    - → ^{16}O should be especia table
- other shell closures occur at 20, 28, 5 , and 126 nucleons
  - unusually large numbers of isot s and isotones due to enhanced stabilit
- A few stable nuclei have both closed neutr nd proton shells
  - very strongly bound (relative to their neighbors)
    - → ^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb
- doubly closed shell nuclei have been synthesized outside stable range
  - ^{56}Ni, ^{100}Sn and ^{132}Sn (unstable)



Filling Example

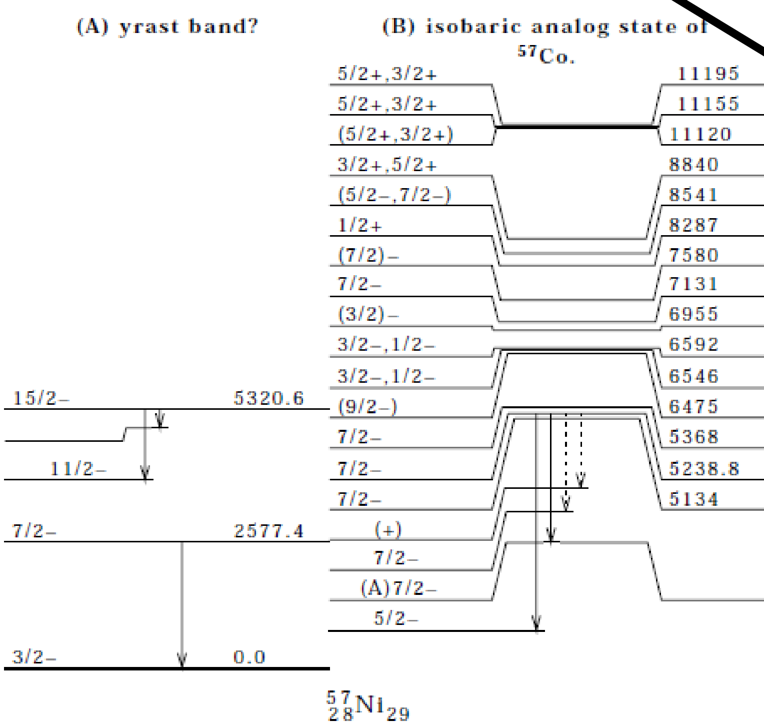
- Consider isotope ^{7}Li
  - 3 protons and 4 neutrons
    - → 2 protons in $1s_{1/2}$, 1 proton in $1p_{3/2}$
    - → 2 neutrons in $1s_{1/2}$, 2 neutrons in $1p_{3/2}$
- spin and angular momentum based on unpaired proton
- spin should be 3/2
- nuclear parity should be negative
  - parity of a p-state (odd l value, l=1)
- Excited state for ^{7}Li?
  - Proton from $1p_{3/2}$ to $1p_{1/2}$
    - → Breaking paired nucleons requires significant energy, neutrons remain paired
  - Bound excited state corresponds to promotion of proton
  - $1p_{1/2}$ corresponds to 1/2-

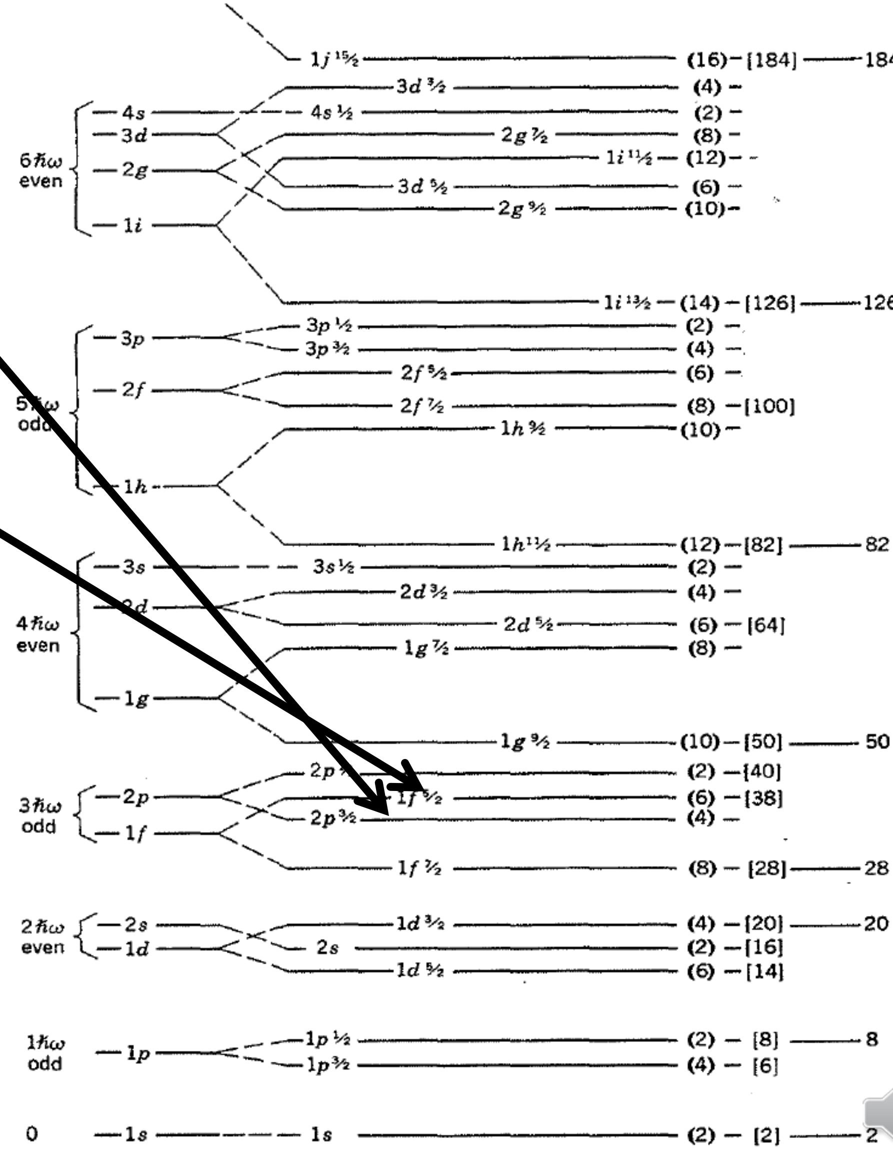




Filling Example

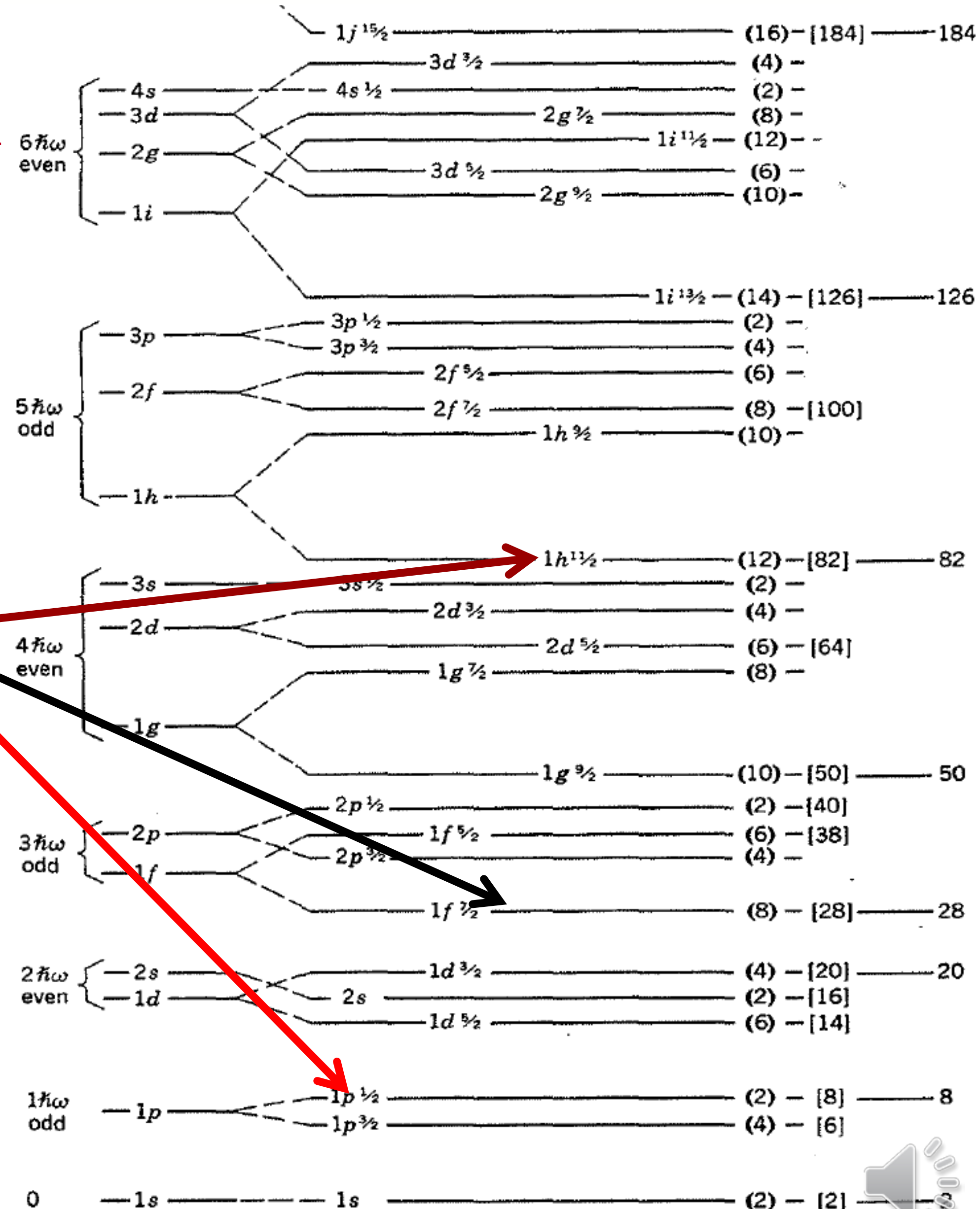
- Consider ^{57}Ni
  - 28 protons, 29 neutrons
    - → Protons fill to $1f_{7/2}$, all paired
    - → Single neutron in $2p_{3/2}$
      - * 3/2– spin and parity
- Excited state of ^{57}Ni
  - From $2p_{3/2}$ to $1f_{5/2}$





Filling Levels

- consider ^{13}C
  - 7th neutron is unpaired
  - p ½ state
    - →½-
- ^{51}V unpaired nucleon is 23rd proton, f 7/2
    - →7/2-
- Not always so straight forward
  - examine ^{137}Ba
    - →81st neutron is unpaired, h 11/2
    - →spin 11/2-
    - →measured as 3/2+
- high spin does not appear as ground
- Deformation impacts level filling

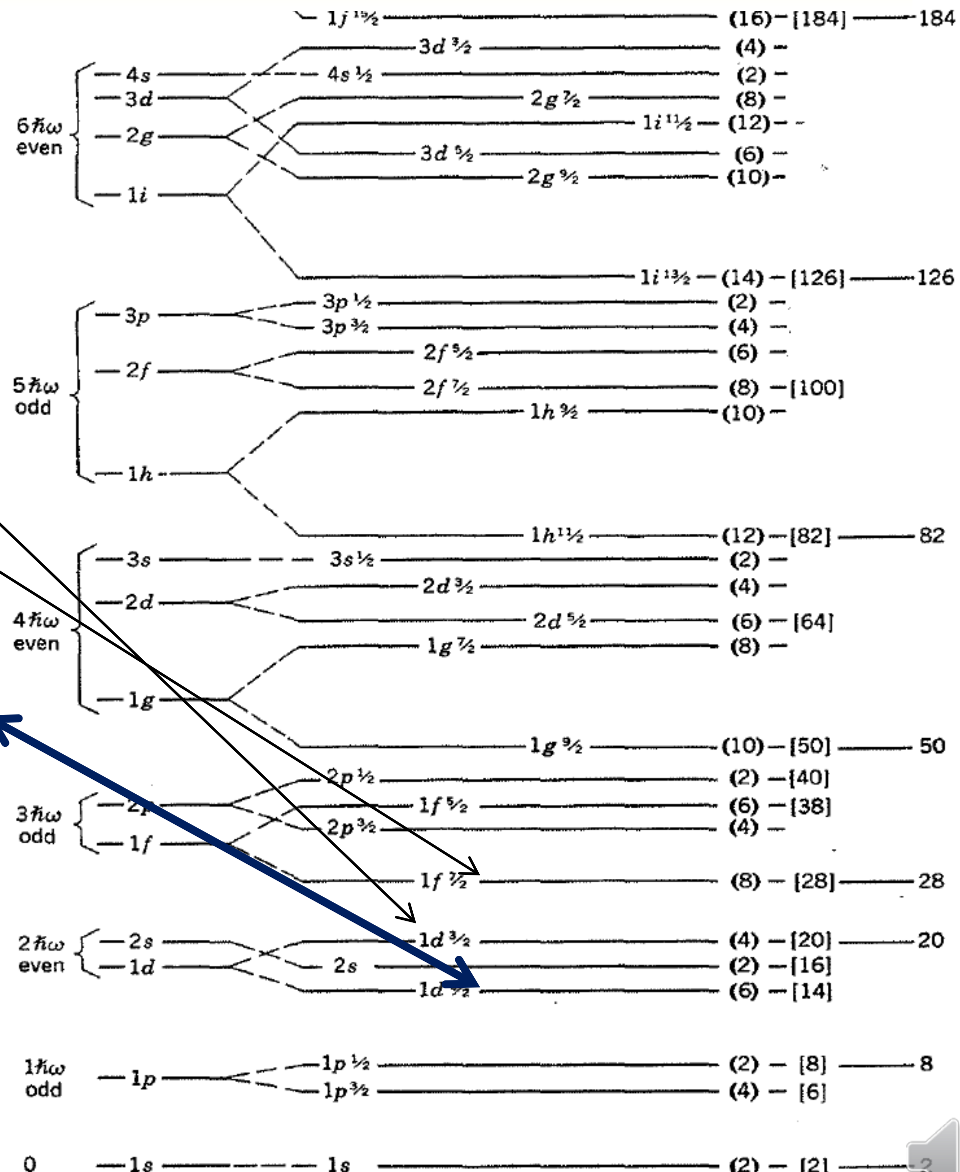


Shell Filling: Spin and parity for odd-odd nuclei

- Configurations with both odd proton and odd neutron have coupling rules to determine spin
  - Integer spin value
- Determine spin based on Nordheim number N
  - Nordheim number N ($=j_1+j_2+\ell_1+\ell_2$) is even, then $I=|j_1-j_2|$
- if N is odd, $I=|j_1 \pm j_2|$
- Parity from sum of l states
  - Even positive parity
  - Odd negative parity
- prediction for configurations in which there is combination of particles and holes is $I=j_1+j_2-1$
- Examples on following page

Shell Model Example

- Consider ^{38}Cl
  - 17 protons (unpaired p in $1d_{3/2}$)
    - → l=2 (d state) and j=3/2
  - 21 neutrons (unpaired n in $1f_{7/2}$)
    - → l=3 (f state) and j=7/2
    - → N= 2+3/2+3+7/2 = 10
    - → Even; I=$|j_1-j_2|$
    - → Spin = 7/2-3/2=2
    - → Parity from l (3+2)=5 (odd), negative parity
  - 2-
- Consider ^{26}Al (13 each p and n)
  - Hole in $1d_{5/2}$, each j = 5/2, each l=2
  - N=5/2+5/2+2+2=9
  - N=odd; I=$|j_1 \pm j_2|$
  - I = 0 or 5 (5 actual value)
  - Parity 2+2=4, even, +
  - 5+



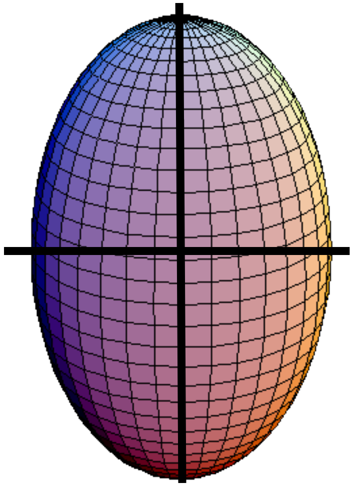
Particle Model: Collective Motion in Nuclei

- Effects of interactions not included in shell-model description
  - pairing force
  - lack of spherically symmetric potential
- Nonspherical Potential
  - intrinsic state
    - → most stable distribution of nucleons among available single-particle states
  - since energy require for deformation is finite, nuclei oscillate about their equilibrium shapes
    - → Deformities 150 <A<190 and A<220
      - * vibrational levels
  - nuclei with stable nonspherical shape have distinguishable orientations in space
    - → rotational levels
    - → polarization of even-even core by motion of odd nucleon
- Splitting of levels in shell model
  - Shell model for spherical nuclei
- Deformation parameter ε_2

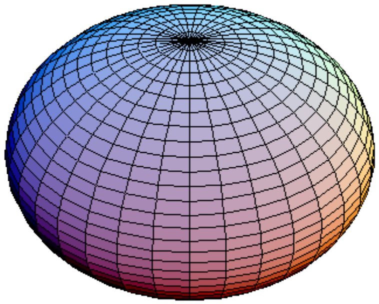
ΔR=major-minor axis
Prolate ΔR is positive
Oblate ΔR is negative

$$\varepsilon_2 = \delta + \frac{1}{6}\delta^2 + \frac{5}{18}\delta^3 + \frac{37}{216}\delta^4 + \ldots$$

$$\delta = \frac{\Delta R}{R_{r.m.s.}}$$



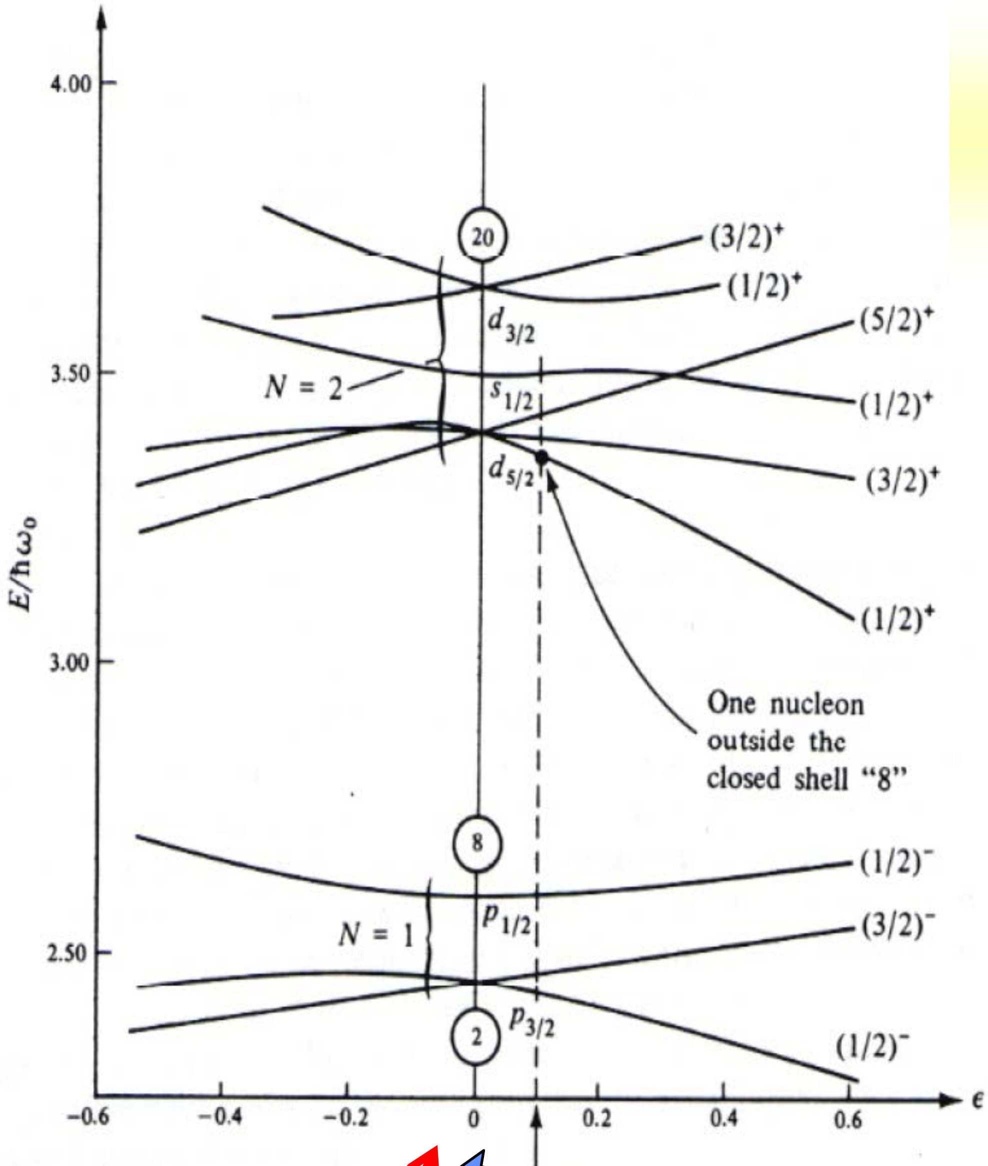
Prolate: polar axis greater than equatorial diameter

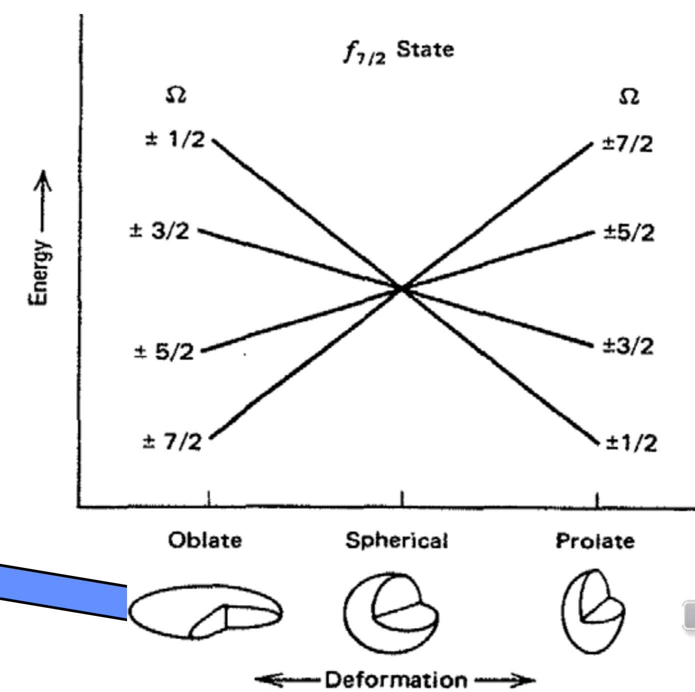


Oblate: polar axis shorter than diameter of equatorial circle

Shell change with deformation

- **Energy of a single nucleon in a deformed potential as a function of deformation ε.**
- **diagram pertains to either Z < 20 or N < 20. Each state can accept two nucleons**





$$\varepsilon_2 = \delta + \frac{1}{6}\delta^2 + \frac{5}{18}\delta^3 + \frac{37}{216}\delta^4 + \ldots$$

Nilsson Diagram

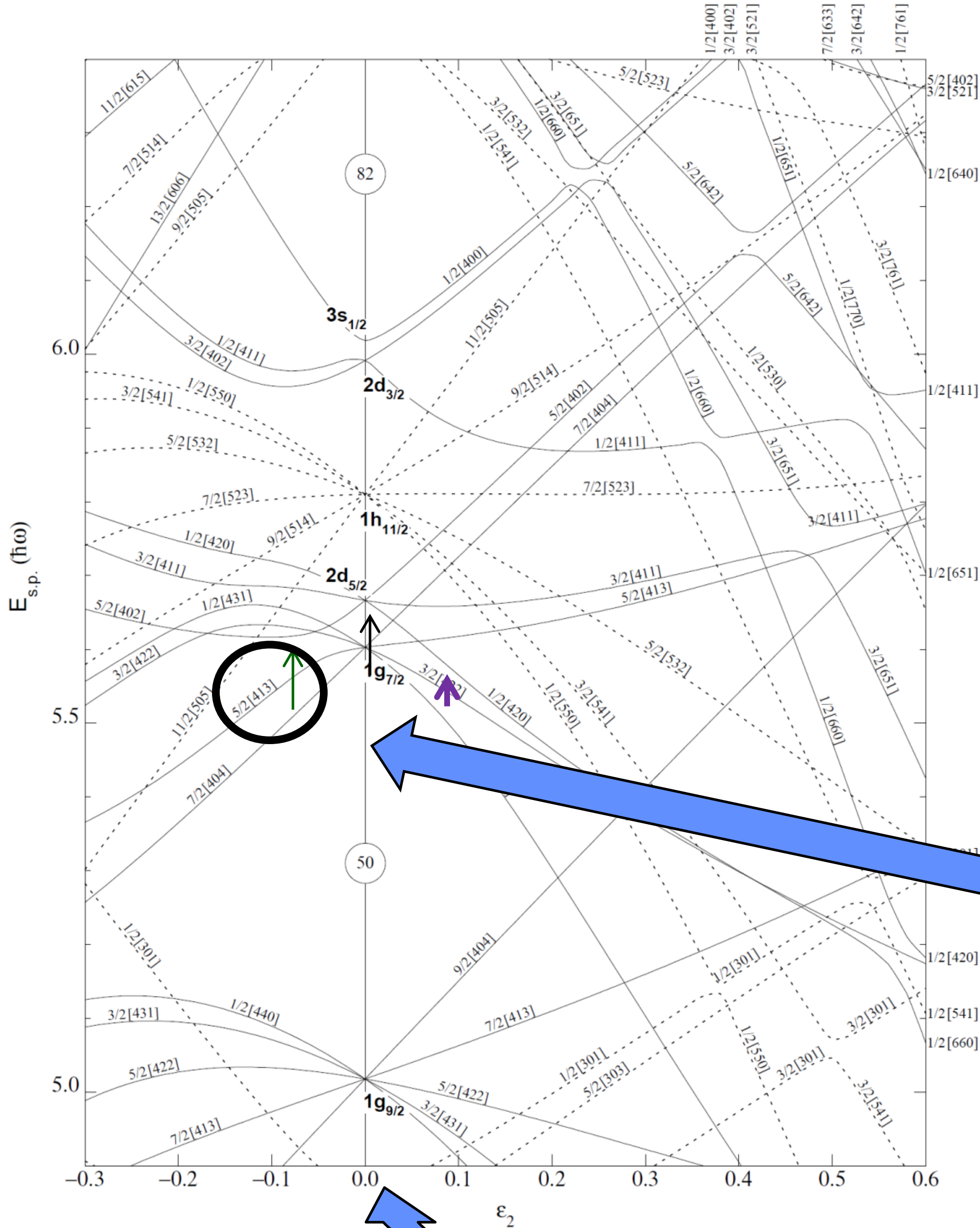


Figure 12. Nilsson diagram for protons, 50 ≤ Z ≤ 82 ($\varepsilon_4 = -\varepsilon_2^2/6$).

- $50 \leq Z \leq 82$
- ^{127}I
  - 53^{rd} proton is unpaired
    - → 7/2+ from shell model
  - measured as 5/2+
- Deformation parameter should show 5/2, even l
  - Oblate nuclei

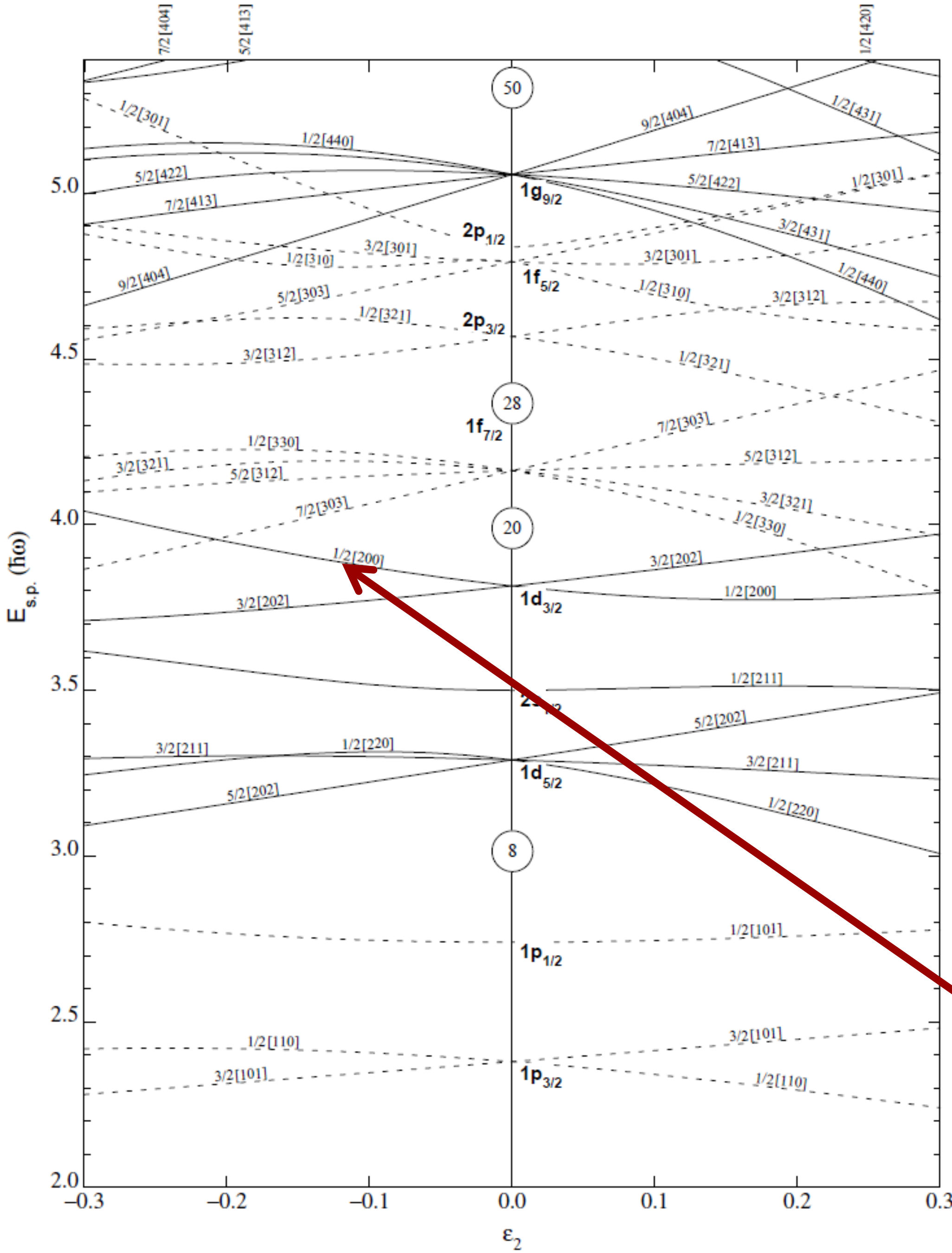
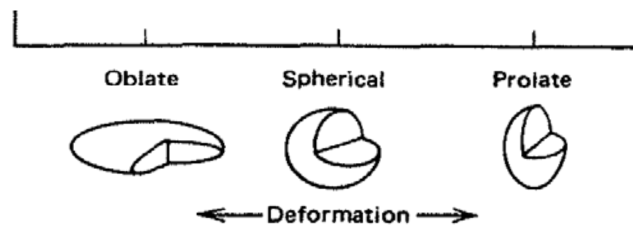
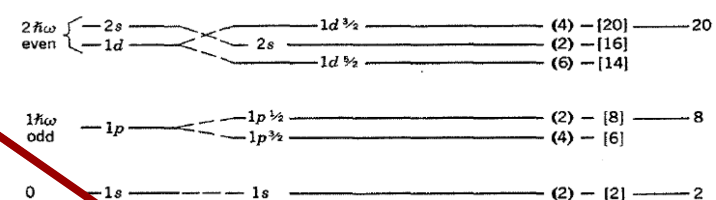


Figure 4. Nilsson diagram for protons or neutrons, Z or N ≤ 50 (ε_4 = 0).

Consider for K isotopes

Which K odd A isotope may be non-spherical?
Z=19





^{47}K ½+

Questions

- What is a nuclear potential
- What are the concepts behind the shell model
- What can be inferred from deviations in spin and parity from the shell model?
- How do nuclear shapes relate to quadrupole moments

Pop Quiz

- **Provide comments on blog**
- **Respond to PDF Quiz 5**